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T. Y. B. Sc. (Sem.-V) (NCF-NEP) Examination October - 2025

Mathematics Major-I MH MJ-501- Group Theory-I

Time: 1 Hour] **[Total Marks: 25**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the questions.

(4) Follow usual notations.

Seat No.:

Student's Signature

Q.1. Answer the following as directed: (05)

(1) Every element a in a group G has the unique inverse in G.

(2) Prove that every cyclic group is always abelian.

(3) For a subgroup H of a group G; define:

(i) Relation of *"Congruence modulo H* ($\equiv$ *mod H*)" on G;

(ii) Right Coset of H in G.

(4) If G is a finite group and a in G, then prove that $a^{o(G)} = e$.

(5) Define Normal Subgroup. Give an illustration of a normal subgroup of a non-abelian group.

Q.2. Attempt any TWO: (10)

(a) Prove; in a group G; that:

(i) If $(a \cdot b)^2 = a^2 \cdot b^2$; $\forall\ a, b$ in G, then G is abelian.

(ii) $(a \cdot b)^{-1} = b^{-1}. a^{-1}$; $\forall\ a, b \in G$.

(b) Prove that $G = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \mid a, b \in \mathbb{R}; a^2 + b^2 \neq 0 \right\}$ is a group under matrix multiplication.

(c) Prove that the intersection of any two subgroups of a group is a subgroup of a group.

Q.3. Attempt any TWO: (10)

(a) Prove that a finite group of prime order is cyclic.

(b) Let H and K be subgroups of a group G. If HK is a subgroup of G, then prove that $HK = KH$.

(c) Let N be subgroup of a group G. Then prove that N is a normal subgroup of G if and only if $g \cdot N \cdot g^{-1} = N$; $\forall\ g \in G$.
